HW 3

1. Give a counterexample to the mean value theorem for vector-valued functions and prove the several variable MVT for scalar valued functions on convex open sets.

2. Suppose the matrix-valued function $[F] : \mathbb{R}^4 \to Mat_{4\times 4} = \mathbb{R}^{16}$ is smooth and its image lies in the set of skew-symmetric matrices. Prove that in general, there is no smooth $g : \mathbb{R}^4 \to \mathbb{R}^4$ such that $[F]_{ij} = \frac{\partial g_i}{\partial x_j} - \frac{\partial g_j}{\partial x_i}\ \forall\ i, j$.

3. If $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable and $f(0) = 0$, then prove that there exist functions $g_i : \mathbb{R}^n \to \mathbb{R}$ such that $f(x) = \displaystyle\sum_{i=1}^{n} x_i g_i(x)$.

4. Are the mixed second partials equal for $f(x, y) = \frac{xy(x^2-y^2)}{x^2+y^2}$ when $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.