HW 6

1. Let $f, g_1, g_2, \ldots, g_k : U \subset \mathbb{R}^n \to \mathbb{R}$ be C^1 functions (where U is open and $k < n$). Suppose f attains a global extremum at $a \in U$ subject to the constraints $g_1 = g_2 = \ldots = g_k = 0$. Assume that $\nabla g_1(a), \nabla g_2(a), \ldots, \nabla g_k(a)$ are linearly independent. Prove that $\nabla f(a) = \lambda_1 \nabla g_1(a) + \lambda_2 \nabla g_2(a) + \ldots$.

2. Prove that if $a, b \geq 0$ and $\frac{1}{p} + \frac{1}{q} = 1$ ($p, q > 0$ real numbers), then $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$ using Lagrange's multipliers.

3. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a C^r (where $r \geq 1$) function. Suppose $f^{-1}(0) \neq \phi$ and whenever $x \in f^{-1}(0)$, $\nabla f(x) \neq 0$. Then prove that $f^{-1}(0)$ is a C^r-manifold with dimension $n - 1$.