

## HW 10

1. Prove rigorously that  $\int_{-\infty}^{\infty} e^{-x^2} dx$  exists as an improper integral and equals  $\sqrt{\pi}$ .
2. Given a smooth manifold-without-boundary  $M \subset \mathbb{R}^n$  and two points  $p \neq q$ , prove that there exist continuous functions  $f, g : M \rightarrow \mathbb{R}$  such that
  - (a)  $f(p) = g(q) = 0, f(q) = g(p) = 1$ .
  - (b) For any coordinate parametrisation  $\alpha$ ,  $f \circ \alpha$  and  $g \circ \alpha$  are smooth.