HW 11

1. Show that the closed upper hemisphere in n-dimensions is an $n - 1$-dimensional smooth manifold-with-boundary whose boundary is the "boundary" $n-1$-dimensional sphere.

2. Prove that the set of orthogonal 3×3 matrices of determinant 1 (denoted as $SO(3)$) is a compact manifold-without-boundary in $\mathbb{R}^9 = Mat(3 \times 3, \mathbb{R})$. What is its dimension?

3. Prove that the closed upper hemisphere in n-dimensions is an orientable manifold.

4. Suppose M is a compact manifold-with-boundary of dimension n in $\mathbb{R}^n$. Show that the topological and manifold interiors coincide.

5. (Optional) Complete the sketch of proof of Green's theorem.