HW 1

1. (Hoffman and Kunze) Suppose R and R' are 2×3 row echelon matrices and that $RX = 0$ and $R'X = 0$ have exactly the same solutions. Prove that $R = R'$.

2. Let $A = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 0 \end{pmatrix}$ be a matrix over $\mathbb{Z}_5$. Solve $AX = Y$ over $\mathbb{Z}_5$ where $Y = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ using row operations.

3. (Hoffman and Kunze) Let C be a 2×2 matrix over a field. Prove that there exist 2×2 matrices A and B such that $C = AB - BA$ iff $C_{11} + C_{22} = 0$.

4. (Hoffman and Kunze) If over a field, A is an $m \times n$ matrix and B is an $n \times m$ matrix where $n < m$, then prove that AB is not invertible.

5. (Hoffman and Kunze) Let A be an $m \times n$ matrix over a field. Show that by using finitely many row and/or column operations, one can bring A to a matrix that is simultaneously row echelon and column echelon.

6. (Hoffman and Kunze) On $\mathbb{R}^2$, define the operations $+((a, b), (c, d)) = (a + b, 0)$ and $.(c, (a, b)) = (ca, 0)$. Which of the axioms of a vector space does this set satisfy ?