HW 4

1. (Hoffman and Kunze) If W is the space of $n \times n$ matrices over the field $\mathbb{F}$ and if f is a linear functional on W such that $f(AB) = f(BA)$ for each $A, B \in W$, then prove that f is a multipler of the trace function.

2. Prove that tensor products are associative.

3. (Hoffman and Kunze) Let A be a 2×2 matrix over a field $\mathbb{F}$ and suppose that $A^2 = 0$. Show that for each scalar c that $\det(cI - A) = c^2$.

4. Show that if $char(\mathbb{F}) \neq 2$, an alternating multilinear map

   $$T : V \times V \times \ldots (arbitrary\ number\ of\ times) \to \mathbb{F}$$

   is equivalent to satisfying the condition that $T(v_{\sigma(1)}, \ldots) = (-1)^{sgn(\sigma)} T(v_1, \ldots)$.

5. Find the determinant of $A = \begin{bmatrix} 1 & x_1 & x_1^2 & \ldots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \ldots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix}$ (The Vandermonde Determinant), where $x_i \in \mathbb{F}$.