MA 229/MA 235 - Lecture 6

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Recap

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- Lagrange's multipliers.

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- Topological manifolds.

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- Definition of smooth manifolds.

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