MA 229/MA 235 - Lecture 1

Vamsi Pritham Pingali

IISc

Logistics

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- TA: Ramesh Mete (rameshmete@iisc.ac.in)
- Text book: Introduction to Smooth Manifolds by John Lee.

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  6. Prove that there are at most finitely many solutions to $a^n + b^n = c^n$ when $n \geq 4$ (a consequence of the Mordell conjecture of Number Theory).

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Review of multivariable calculus - The topology of $\mathbb{R}^n$

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Review of multivariable calculus - The topology of $\mathbb{R}^n$

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Review of multivariable calculus - Continuity

Review of multivariable calculus - Continuity

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- The usual laws

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Review of multivariable calculus - Continuity

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- The usual laws of continuity hold. Hence, rational functions are continuous wherever the denominator is not zero.
- Invariance of domain: If U is open, and $F : U \subset \mathbb{R}^n \to \mathbb{R}^n$ is $1 - 1$ and continuous, then $F(U)$ is open and U is homeomorphic to it. As a consequence, an open subset of $\mathbb{R}^n$ cannot be homeomorphic to an open subset of $\mathbb{R}^m$ when $m \neq n$. The proof uses

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Review of multivariable calculus - Continuity

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Review of multivariable calculus - differentiability

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