

HW 1

1. Let $f(x, y) = \frac{xy^2}{x^2+y^4}$ if $x \neq 0$ and $f(0, y) = 0$. Prove that f is directionally differentiable along all directions at $(0, 0)$ but is not differentiable at $(0, 0)$.
2. Let $f(x, y) = ||x| - |y|| - |x| - |y|$. Prove that the partials exist and f is continuous at $(0, 0)$ but it isn't differentiable there.
3. Prove that a symmetric 2×2 matrix S (with real entries) is positive definite if and only if (i) the diagonal entries of S are positive and (ii) $\det(S) > 0$.
4. Find the critical points of $f(x, y) = 4x^2 + 9y^2 + 8x - 36y + 24$ and determine whether they are local maxima, minima or saddle points.