HW

Let (X, d) be a metric space and $A \subset X$ be nonempty.

1. Show that $d(x, A) = 0$ if and only if $x \in \bar{A}$

2. Show that if A is compact, $d(x, A) = d(x, a_x)$ for some $a_x \in A$.

3. Define the ϵ-neighbourhood of A in X to be the set $U(A, \epsilon) = \{x | d(x, A) < \epsilon\}$. Show that $U(A, \epsilon)$ equals the union of the open balls $B_d(a, \epsilon)$ for $a \in A$.

4. Assume that A is compact; let $A \subset U$ and U be open. Show that some ϵ-neighbourhood of A is contained in U.

5. Show that the result in the previous part need not hold if A is closed but not compact.