HW 5 (to be submitted by Mar 8)

1. (10 marks) Let M be a compact n-dimensional manifold without boundary. Cover M with finitely many coordinate charts $(U_1, \phi_1 = \vec{x}_1), (U_2, \phi_2 = \vec{x}_2), \ldots, (U_k, \phi_k = \vec{x}_k)$. Let ρ_j be a smooth partition-of-unity subordinate to this open cover. Define $F : M \to \mathbb{R}^{nk+k}$ by $F(p) = (\rho_1(p)\phi_1(p), \ldots, \rho_k(p)\phi_k(p), \rho_1(p), \ldots, \rho_k(p))$. Prove that F is an embedding.

2. (5 marks) Let $S \subset M$ be an embedded submanifold and M be a manifold. Let N be a manifold. Then if $F : N \to M$ is a smooth map such that $F(N) \subset S$, then $F : N \to S$ is a smooth map.

3. (10 marks) Assume the fact that if $G : U \subset \mathbb{R}^n \to \mathbb{R}^n$ is a smooth map, then $G(\textit{measure zero}) = \textit{measure zero}$. Now prove that if $F : M \to N$ is a smooth map and $\dim(M) < \dim(N)$, then the critical value set S is measure zero in N, i.e., suppose $\phi : U \to \mathbb{R}^{\dim(N)}$ is a coordinate chart, then $\phi(U \cap S)$ has measure zero in $\mathbb{R}^{\dim(N)}$. (Hint: Use an appropriately defined map $H : M \times \mathbb{R}^{\dim(N)-\dim(M)} \to N$.)