HW 8 (to be submitted by April 5)

1. (7 marks) Let U be a bounded open convex subset of $\mathbb{R}^m$ and $G : \bar{U} \to \mathbb{R}^n$ be a smooth map. Prove that
   (a) (3 marks) $\|G(x) - G(y)\| \leq C\|x - y\|$.
   (b) (4 marks) Prove that if $m \leq n$ and $E \subset U$ has measure zero, then $G(E)$ has measure zero.
2. (8 marks) Prove that a submanifold of $\mathbb{R}^n$ of dimension at most $n - 1$ has measure zero in $\mathbb{R}^n$.
3. (10 marks) Prove that $\int_{-\infty}^{\infty} e^{-x^2} dx$ (in the Lebesgue sense) is $\sqrt{\pi}$ using the strategy described in the class (using multivariable calculus). You are allowed to use the change of variables formula mentioned in the class and measure theory.