HW 1 (due on Aug 16, Wednesday, in the class)

Please write your answers clearly and rigorously. Write your name in plain lettering (as opposed to cursive) and also staple all the pages.

1. (Spivak, chapter 1, problem 8 (suitably modified)) For this problem assume

    (a) (The generalised Jordan curve theorem) If $A \subset \mathbb{R}^n$ is homeomorphic to S^{n-1}, then $\mathbb{R}^n - A$ has 2 components, and A is the boundary of each.

    (b) If $B \subset \mathbb{R}^n$ is homeomorphic to $D^n = \{x \in \mathbb{R}^n : d(x, 0) \leq 1\}$, then $\mathbb{R}^n - B$ is connected.

    Prove that

    (a) One component of $\mathbb{R}^n - A$ (the "outside of A") is unbounded, and the other (the "inside of A") is bounded.

    (b) If $U \subset \mathbb{R}^n$ is an open ball of radius 2ϵ centred at p, $A \subset U$ is the sphere S^{n-1} of radius ϵ centred at p, and $f : U \to \mathbb{R}^n$ is 1-1 and continuous, then $f(inside\ of\ A) =\ inside\ of f(A)$.

    (c) Prove Invariance of Domain.

2. (Spivak, chapter 2, problem 4) How many distinct $\mathbb{C}^\infty$ structures are there on $\mathbb{R}$? (There is only one upto diffeomorphism, but that is not the question asked) I just want to know whether the answer is finite or infinite, not the cardinality.