HW 3 (due on Sept 1, Friday, in the class)

Please write your answers clearly and rigorously. Write your name in plain lettering (as opposed to cursive) and also staple all the pages.

Let M be a smooth manifold. Let $\{U_\alpha\}$ be an open cover of M. Define

$$V = \frac{\coprod_\alpha U_\alpha \times \mathbb{R}^r}{(p, \vec{v}_\alpha) \sim (p, g_{\alpha\beta}(p)\vec{v}_\beta)}$$

where $g_{\alpha\beta} : U_\alpha \cap U_\beta \to GL(r)$ is a collection of smooth matrix-valued functions satisfying (for all α, β)

1. $g_{\alpha\alpha} = I$.
2. $g_{\alpha\beta} = g_{\beta\alpha}^{-1}$, and
3. $g_{\alpha\beta} g_{\beta\gamma} g_{\gamma\alpha} = I$.

Prove that

1. $\sim$ is an equivalence relation.
2. V is a smooth rank-r real vector bundle over M. The $g_{\alpha\beta}$ are called "transition functions of the vector bundle V". (The terminology is a little confusing because we used it for a somewhat different reason earlier.)
3. Every smooth rank-r real vector bundle over M is isomorphic to V for some (appropriate) choice of $\{U_\alpha\}, g_{\alpha\beta}$.