

HW 5 (due on Sept 15, Friday, in the class)

Please write your answers clearly and rigorously. Write your name in plain lettering (as opposed to cursive) and also staple all the pages.

1. Suppose M is a connected smooth manifold. Given any two points p, q prove that there is a diffeomorphism taking p to q . (Hint : Prove that the set of all points connected to p by diffeomorphisms is non-empty, open, and closed.)
2. Consider the (infinite) Möbius strip thought of as a vector bundle of rank-1 over the circle (using a problem from the previous HW with $T = -1$). By the way, vector bundles of rank-1 are called “line bundles”. Prove that the Möbius line bundle is not isomorphic to the trivial bundle $S^1 \times \mathbb{R}$.