HW 7 (due on Oct 23, Monday, in the class)

Please write your answers clearly and rigorously. Write your name in plain lettering (as opposed to cursive) and also staple all the pages.

1. (Spivak chapter 4, problem 2) : If $f, g : M \to \mathbb{R}$ are smooth, then prove that $d(fg) = dfg + gdf$.

2. (Spivak chapter 10, problem 4)) : Let G be a connected topological group (basically a topological space which is a group where multiplication and inversion are continuous), and U be any neighbourhood of the identity e. Let U^n denote the set of all products $a_1 \ldots a_n$ where $a_i \in U$.

   (a) Show that U^{n+1} is a neighbourhood of U^n.

   (b) Conclude that $\cup_n U^n = G$.

3. If $AB = BA$ then $e^{A+B} = e^A e^B$ for two $n \times n$ real matrices A, B.

4. (Spivak chapter 4, problem 5) This is an OPTIONAL problem that I will not grade. But, it gives you a lot of practice with tensors and the index notation.