HW 8 (due on Nov 8, Wednesday, in the class)

Please write your answers clearly and rigorously. Write your name in plain lettering (as opposed to cursive) and also staple all the pages.

1. Suppose V and W are smooth vector bundles over a manifold M. Prove that smooth bundle maps $T : V \to W$ naturally correspond to smooth sections of $W \otimes V^*$.

2. (Spivak chapter 7, problem 5) Show that m functions $f_1, \ldots, f_m : M \to \mathbb{R}$ form a coordinate system in a neighbourhood of p if and only if $df_1(p) \wedge df_2(p) \wedge \ldots df_m(p) \neq 0$.

3. (Spivak chapter 7, problem 6) An element $\omega \in \Omega^k$ is called decomposable if $\omega = \phi^1 \wedge \phi^2 \ldots \phi^k$ for some $\phi^i \in V^*$. Prove that if

   (a) $dim(V) \leq 3$, then every $\omega \in \Omega^2$ is decomposable.

   (b) Come up with a counterexample in $dim(V) = 4$ of a 2-form that is not decomposable.