HW 9 (due on Nov 22, Wednesday, in the class)

Please write your answers clearly and rigorously. Write your name in plain lettering (as opposed to cursive) and also staple all the pages.

1. Suppose ω is a smooth 1-form (field) on a smooth manifold M. Suppose X is a smooth vector field on M. Prove that

   (a) $L_X\omega$ is a smooth 1-form.

   (b) $i_X d\omega + d(i_X\omega) = L_X\omega$ where i_X is the so-called contraction operation defined as follows : Suppose η is a k-form, then $i_X\eta$ is the $k-1$ form satisfying $i_X\eta(X_1,\ldots,X_{k-1}) = \eta(X,X_1,\ldots,X_{k-1})$. (It turns out that the notion of $L_X\omega$ can be defined for k-forms and $di_X + i_X d = L_X$ holds for them too. This is called "Cartan's magic formula".)

2. (Spivak Chapter 8, Problem 20) Prove that a non-compact connected manifold is that union $U_1 \cup U_2 \ldots$ where U_i are coordinate neighbourhoods with $U_i \cap U_{i+1} \neq \phi$, and the sequence is eventually outside of any compact set.