HW 1 (due on 18th January (Thursday) in the class)

Please write your answers clearly and rigorously. Write your name in plain lettering (as opposed to cursive) and also staple all the pages.

1. Prove that $C^{k,\alpha}(\bar{U})$ is a Banach space.
2. (Problem 17, chapter 5 in Evans) Assume that $F : \mathbb{R} \to \mathbb{R}$ is C^1, with F' bounded. Suppose U is bounded and $u \in W^{1,p}(U)$ for some $1 \leq p < \infty$. Show that $v = F(u)$ is in $W^{1,p}(U)$ and $v_{x_i} = F'(u)u_{x_i}$.
3. Show that if $u, v \in H^s(S^1 \times S^1 \ldots)$ where $s > \frac{n}{2}$, then $uv \in H^s(S^1 \times S^1 \ldots)$ and $\|uv\|_{H^s} \leq C\|u\|_{H^s}\|v\|_{H^s}$.