HW 5 (due on 5th April (Thursday) in the class)

Please write your answers clearly and rigorously. Write your name in plain lettering (as opposed to cursive) and also staple all the pages.

1. Suppose E is a vector bundle on a compact manifold M. Cover M with finitely many coordinate trivialising neighbourhoods U_μ such that $\bar{U}_\mu$ is still contained in a coordinate trivialising neighbourhood. Let u be a section of E and $u = u^i_\mu e_{i,\mu}$ be the local representation on U_μ. Define $C^{k,\alpha}$ as the space of C^k sections u such that $u_\mu \in C^{k,\alpha}(\bar{U}_\mu)$ and the norm $\|u\|_{C^{k,\alpha}} = \sum\limits_{\mu} \|u_\mu\|_{C^{k,\alpha}}$. Prove that this is a Banach space and that if you make different choices of U_μ and the trivialisations and coordinates, you get a quasi-isometric Banach space.

2. Prove that

   (a) A section v of E is in H^{-s} (where $s \geq 0$ is an integer) if and only if $\rho_\mu v \in H^{-s}(S^1 \times S^1 \ldots, \mathbb{R}^r)$.

   (b) Prove that the H^{-s} norm is equivalent to $\sum_\mu \|\rho_\mu v\|_{H^{-s}(S^1 \times S^1 \ldots)}$ where ρ_μ is a partition-of-unity.