Lecture 16 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

IISc

Recap

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- Defined Hermitian and Skew-Hermitian

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- Proved that they had real eigenvalues and that they could be diagonalised using unitary matrices.

An ODE to a Grecian urn

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- The same result works for $\mathbb{R}$.

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- If $A = A^T$ then by the Spectral Theorem A is diagonalisable by an orthogonal matrix. In that case, the above strategy applies.

Example

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- Solve for all differentiable functions

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- The matrix A is $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$.

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- Now $\begin{bmatrix} x \\ y \end{bmatrix} = Pw = ae^{t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + be^{2t}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

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