Lecture 18 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

IISc

Recap

Recap

- Proved that

Recap

- Proved that $y' = ky$ on $\mathbb{R}$

Recap

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Recap

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- Studied a method

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- Studied a method to solve *systems*

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- Recast $y'' + Py' + Qy = 0$ where $P, Q \in \mathbb{R}$ are constants as a system of first-order ODE. Solved it in the case where the roots of $D^2 + PD + Q = 0$ real and distinct.

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- If we insist on *real* solutions then $\bar{y} = y$. Thus (why?) $y(t) = e^{\alpha t}(A\cos(\beta t) + B\sin(\beta t))$ where $A, B \in \mathbb{R}$.

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- Consider $y = e^{\lambda t}(A + Bt)$. Clearly this is two-dimensional and a solution. Thus it is *the* solution.

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- Proof: Let $g(x) = \int_{x_0}^{x} P(t)dt$.

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- Proof: Let $g(x) = \int_{x_0}^{x} P(t)dt$. By the FTC g is differentiable on (a, b) and $g'(x) = P(x)$.

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- Proof: Let $g(x) = \int_{x_0}^{x} P(t)dt$. By the FTC g is differentiable on (a, b) and $g'(x) = P(x)$. Note that $(ye^{g(x)})'e^{-g(x)} = y' + yg'(x) = y' + P(x)y = 0$.

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- Proof: Let $g(x) = \int_{x_0}^{x} P(t)dt$. By the FTC g is differentiable on (a, b) and $g'(x) = P(x)$. Note that $(ye^{g(x)})'e^{-g(x)} = y' + yg'(x) = y' + P(x)y = 0$. Hence $ye^{g(x)}$ is a constant.

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- Theorem: Let $P : (a, b) \to \mathbb{R}$ be continuous. Let $x_0 \in (a, b)$. There exists a unique differentiable function $y : (a, b) \to \mathbb{R}$ satisfying the ODE above and $y(x_0) = A$ where $A \in \mathbb{R}$ is given. Moreover, $y(x) = Ae^{-\int_{x_0}^{x} P(t)dt}$.
- Proof: Let $g(x) = \int_{x_0}^{x} P(t)dt$. By the FTC g is differentiable on (a, b) and $g'(x) = P(x)$. Note that $(ye^{g(x)})'e^{-g(x)} = y' + yg'(x) = y' + P(x)y = 0$. Hence $ye^{g(x)}$ is a constant. Thus $y = Ae^{-\int_{x_0}^{x} P(t)dt}$. □

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An example of a separable nonlinear first-order ODE

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