Lecture 19 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

IISc

Recap

Recap

- Solved second-order

Recap

- Solved second-order linear homogeneous

Recap

- Solved second-order linear homogeneous ODE with constant-coefficients.

Recap

- Solved second-order linear homogeneous ODE with constant-coefficients.
- Solved first-order

Recap

- Solved second-order linear homogeneous ODE with constant-coefficients.
- Solved first-order linear ODE.

Recap

- Solved second-order linear homogeneous ODE with constant-coefficients.
- Solved first-order linear ODE.
- Made some remarks about

Recap

- Solved second-order linear homogeneous ODE with constant-coefficients.
- Solved first-order linear ODE.
- Made some remarks about nonlinear separable ODE.

Recap

- Solved second-order linear homogeneous ODE with constant-coefficients.
- Solved first-order linear ODE.
- Made some remarks about nonlinear separable ODE.
- Defined equilibria.

Phase diagrams and trajectories (Not for exams)

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$,

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$.

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$,

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ,

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y')

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on.

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.
- A qualitative picture of

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.
- A qualitative picture of the solution is gleaned from

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.
- A qualitative picture of the solution is gleaned from these phase diagrams and trajectories.

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.
- A qualitative picture of the solution is gleaned from these phase diagrams and trajectories.
- For instance,

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.
- A qualitative picture of the solution is gleaned from these phase diagrams and trajectories.
- For instance, one can analyse $x' = ax(1-x) - h(1+\sin(2\pi t))$ that

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.
- A qualitative picture of the solution is gleaned from these phase diagrams and trajectories.
- For instance, one can analyse $x' = ax(1 - x) - h(1 + \sin(2\pi t))$ that represents the

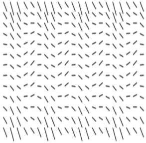
population of fish

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.
- A qualitative picture of the solution is gleaned from these phase diagrams and trajectories.
- For instance, one can analyse $x' = ax(1 - x) - h(1 + \sin(2\pi t))$ that represents the population of fish with periodic harvesting.

Phase diagrams and trajectories (Not for exams)

- If $y' = g(y)$, one can draw tiny lines indicating the slope of a potential solution at every point (x, y) in $\mathbb{R}^2$. These are called phase diagrams. (Note that one can do the same even for $y' = F(x, y)$.)
- If $y(0) = y_0$, one can "flow" along the tiny line at $(0, y_0)$ for a small time δ, and then look at the slope at (δ, y') and so on. These curves are called trajectories of the "vector field" associated to the ODE.
- A qualitative picture of the solution is gleaned from these phase diagrams and trajectories.
- For instance, one can analyse $x' = ax(1 - x) - h(1 + \sin(2\pi t))$ that represents the



population of fish with periodic harvesting.

Euler's method (Not for exams)

Euler's method (Not for exams)

- Suppose we want to solve

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method:

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size.

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$.

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance).

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x + h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately numerically unstable often, i.e.,

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately numerically unstable often, i.e., the errors keep building up.

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately numerically unstable often, i.e., the errors keep building up. There are better methods

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately numerically unstable often, i.e., the errors keep building up. There are better methods like the Runge-Kutta methods (

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately numerically unstable often, i.e., the errors keep building up. There are better methods like the Runge-Kutta methods (that replace

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately numerically unstable often, i.e., the errors keep building up. There are better methods like the Runge-Kutta methods (that replace the slope with a

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately numerically unstable often, i.e., the errors keep building up. There are better methods like the Runge-Kutta methods (that replace the slope with a weighted sum of slopes)

Euler's method (Not for exams)

- Suppose we want to solve $y' = g(y)f(x)$ with $y(0) = y_0$ numerically. (or more generally, $y' = F(x, y)$).
- Using the intuition from phase diagrams, we arrive at Euler's iterative method: $y_{n+1} = y_n + F(x_n, y_n)h$ where h is the step-size. More rigorously, $y(x+h) = y(x) + \int_x^{x+h} F(x(t), y(t))dt$. This formulation of an ODE as an integral equation is useful for computational and theoretical purposes (i.e. to prove existence and uniqueness for instance). Watch Hidden Figures for instance!
- This method is unfortunately numerically unstable often, i.e., the errors keep building up. There are better methods like the Runge-Kutta methods (that replace the slope with a weighted sum of slopes) that yield much better error estimates.

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier,

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution,

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution is $y_1 + c_1 u_1(x) + c_2 u_2(x)$.

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution is $y_1 + c_1 u_1(x) + c_2 u_2(x)$. Hence, we "just"

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution is $y_1 + c_1 u_1(x) + c_2 u_2(x)$. Hence, we "just" need to find *one*

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution is $y_1 + c_1 u_1(x) + c_2 u_2(x)$. Hence, we "just" need to find *one*solution of

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution is $y_1 + c_1 u_1(x) + c_2 u_2(x)$. Hence, we "just" need to find *one*solution of the inhomogeneous equation.

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution is $y_1 + c_1 u_1(x) + c_2 u_2(x)$. Hence, we "just" need to find *one*solution of the inhomogeneous equation. There is a way

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution is $y_1 + c_1 u_1(x) + c_2 u_2(x)$. Hence, we "just" need to find *one*solution of the inhomogeneous equation. There is a way to do this using

Second-order linear inhomogeneous ODE with some constant coefficients (Back to exams)

- Now we attempt to solve $y'' + Py' + Qy = R(x)$ where $P, Q \in \mathbb{R}$ and $R(x)$ is a continuous function.
- As earlier, if y_1 is a *particular* solution and y_2 is any other solution, then $y_1 - y_2$ solves the corresponding homogeneous equation.
- We already proved that the solutions of the homogeneous equation form a real two-dimensional vector space spanned by two functions $u_1(x), u_2(x)$.
- Thus the general solution is $y_1 + c_1 u_1(x) + c_2 u_2(x)$. Hence, we "just" need to find *one*solution of the inhomogeneous equation. There is a way to do this using the homogeneous equation itself.

Variation of parameters

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.
  $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus
  $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.
  $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus
  $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.
  $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus
  $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.
  $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus
  $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.
  $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus
  $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + Pa_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.
  $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus
  $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!)

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.
  $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus
  $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!) This determinant is denoted as

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + Pa_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!) This determinant is denoted as $W(x) = u_1 u_2' - u_2 u_1'$ and is called

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$.
  $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus
  $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!) This determinant is denoted as $W(x) = u_1 u_2' - u_2 u_1'$ and is called the Wronskian of u_1, u_2 (

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!) This determinant is denoted as $W(x) = u_1 u_2' - u_2 u_1'$ and is called the Wronskian of u_1, u_2 (after a crazy Polish chap).

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!) This determinant is denoted as $W(x) = u_1 u_2' - u_2 u_1'$ and is called the Wronskian of u_1, u_2 (after a crazy Polish chap).
- Thus $a_1' = \frac{-u_2 R(x)}{W(x)}$ and

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + Pa_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!) This determinant is denoted as $W(x) = u_1 u_2' - u_2 u_1'$ and is called the Wronskian of u_1, u_2 (after a crazy Polish chap).
- Thus $a_1' = \frac{-u_2 R(x)}{W(x)}$ and $a_2' = \frac{u_1 R(x)}{W(x)}$.

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!) This determinant is denoted as $W(x) = u_1 u_2' - u_2 u_1'$ and is called the Wronskian of u_1, u_2 (after a crazy Polish chap).
- Thus $a_1' = \frac{-u_2 R(x)}{W(x)}$ and $a_2' = \frac{u_1 R(x)}{W(x)}$. Integrating these,

Variation of parameters

- Clearly $y = a_1 u_1 + a_2 u_2$ where a_1, a_2 are constants does *not* solve the inhomogeneous equation. What if we let them vary with x? (Method of variation of parameters.)
- Let's try $y = a_1(x)u_1(x) + a_2(x)u_2(x)$. $y' = \sum_i a_i u_i' + a_i' u_i$. $y'' = \sum_i (a_i' u_i)' + a_i' u_i' + a_i u_i''$. Thus $y'' + Py' + Qy = \sum_i (a_i' u_i)' + a_i' u_i' + P a_i' u_i = R$.
- This is just one equation with two knowns $a_1(x), a_2(x)$. The trick is to set $a_1' u_1 + a_2' u_2 = 0$ and $a_1' u_1' + a_2' u_2' = R$.
- So we have $A \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix}$ where $A = \begin{bmatrix} u_1 & u_2 \\ u_1' & u_2' \end{bmatrix}$.
- Since u_1, u_2 are linearly independent $\det(A) \neq 0$ (?!) This determinant is denoted as $W(x) = u_1 u_2' - u_2 u_1'$ and is called the Wronskian of u_1, u_2 (after a crazy Polish chap).
- Thus $a_1' = \frac{-u_2 R(x)}{W(x)}$ and $a_2' = \frac{u_1 R(x)}{W(x)}$. Integrating these, we get the desired particular solution.

An example

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters:

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$.

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$. The Wronskian of $\cos(x), \sin(x)$ is

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$. The Wronskian of $\cos(x), \sin(x)$ is $W(x) = \cos^2(x) + \sin^2(x) = 1$.

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$. The Wronskian of $\cos(x), \sin(x)$ is $W(x) = \cos^2(x) + \sin^2(x) = 1$.
- Hence $a_1' = -\sin(x)\tan(x)$ and $a_2' = \cos(x)\tan(x) = \sin(x)$.

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$. The Wronskian of $\cos(x), \sin(x)$ is $W(x) = \cos^2(x) + \sin^2(x) = 1$.
- Hence $a_1' = -\sin(x)\tan(x)$ and $a_2' = \cos(x)\tan(x) = \sin(x)$. Thus $a_2 = -\cos(x)$ and

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$. The Wronskian of $\cos(x), \sin(x)$ is $W(x) = \cos^2(x) + \sin^2(x) = 1$.
- Hence $a_1' = -\sin(x)\tan(x)$ and $a_2' = \cos(x)\tan(x) = \sin(x)$. Thus $a_2 = -\cos(x)$ and $a_1 = \int(-\sec(x) + \cos(x)) = \sin(x) - \ln|\sec(x) + \tan(x)|$.

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$. The Wronskian of $\cos(x), \sin(x)$ is $W(x) = \cos^2(x) + \sin^2(x) = 1$.
- Hence $a_1' = -\sin(x)\tan(x)$ and $a_2' = \cos(x)\tan(x) = \sin(x)$. Thus $a_2 = -\cos(x)$ and $a_1 = \int(-\sec(x) + \cos(x)) = \sin(x) - \ln|\sec(x) + \tan(x)|$.
- Therefore

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$. The Wronskian of $\cos(x), \sin(x)$ is $W(x) = \cos^2(x) + \sin^2(x) = 1$.
- Hence $a_1' = -\sin(x)\tan(x)$ and $a_2' = \cos(x)\tan(x) = \sin(x)$. Thus $a_2 = -\cos(x)$ and $a_1 = \int(-\sec(x) + \cos(x)) = \sin(x) - \ln|\sec(x) + \tan(x)|$.
- Therefore the general solution is

An example

- Solve $y'' + y = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$.
- First we solve $y'' + y = 0$. In this case our methods easily produce $y = c_1 \cos(x) + c_2 \sin(x)$.
- Now we attempt variation of parameters: $y_1 = a_1(x)\cos(x) + a_2(x)\sin(x)$. The Wronskian of $\cos(x), \sin(x)$ is $W(x) = \cos^2(x) + \sin^2(x) = 1$.
- Hence $a_1' = -\sin(x)\tan(x)$ and $a_2' = \cos(x)\tan(x) = \sin(x)$. Thus $a_2 = -\cos(x)$ and $a_1 = \int(-\sec(x) + \cos(x)) = \sin(x) - \ln|\sec(x) + \tan(x)|$.
- Therefore the general solution is $y = c_1\cos(x) + c_2\sin(x) - \cos(x)\ln|\sec(x) + \tan(x)|$.

Other methods in special cases: R is a polynomial

Other methods in special cases: R is a polynomial

- The above procedure

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general.

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance,

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$.

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$.

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$.

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$. Thus $y_1 = x^3 - 6x$ is

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$. Thus $y_1 = x^3 - 6x$ is a particular solution.

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$. Thus $y_1 = x^3 - 6x$ is a particular solution. The general solution is

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$. Thus $y_1 = x^3 - 6x$ is a particular solution. The general solution is $y = x^3 - 6x + c_1 \cos(x) + c_2 \sin(x)$.

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$. Thus $y_1 = x^3 - 6x$ is a particular solution. The general solution is $y = x^3 - 6x + c_1 \cos(x) + c_2 \sin(x)$.
- On the other hand,

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$. Thus $y_1 = x^3 - 6x$ is a particular solution. The general solution is $y = x^3 - 6x + c_1 \cos(x) + c_2 \sin(x)$.
- On the other hand, variation of parameters leads

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$. Thus $y_1 = x^3 - 6x$ is a particular solution. The general solution is $y = x^3 - 6x + c_1 \cos(x) + c_2 \sin(x)$.
- On the other hand, variation of parameters leads to annoying integrals like

Other methods in special cases: R is a polynomial

- The above procedure is painful to implement in general. Suppose R is a polynomial of degree n and $Q \neq 0$.
- We can try $y_1 = \sum_{k=0}^{n} a_k x^k$.
- For instance, solve $y'' + y = x^3$. Let's try $y_1 = Ax^3 + Bx^2 + Cx + D$. Then $A = 1, B = 0, C = -6, D = 0$. Thus $y_1 = x^3 - 6x$ is a particular solution. The general solution is $y = x^3 - 6x + c_1 \cos(x) + c_2 \sin(x)$.
- On the other hand, variation of parameters leads to annoying integrals like $\int x^3 \cos(x)$.

Other methods in special cases: $R = p(x)e^{mx}$

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m+P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m+P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m+P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m+P)u' + (m^2+Pm+Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2+Pm+Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m+P$ is not zero,

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m+P)u' + (m^2+Pm+Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2+Pm+Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m+P$ is not zero, then a degree $n+1$ polynomial. If both are zero,

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial. If both are zero, then a degree $n + 2$ polynomial.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial. If both are zero, then a degree $n + 2$ polynomial.
- Find a particular solution

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m+P)u' + (m^2+Pm+Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2+Pm+Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m+P$ is not zero, then a degree $n+1$ polynomial. If both are zero, then a degree $n+2$ polynomial.
- Find a particular solution to $y'' + y = xe^{3x}$.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial. If both are zero, then a degree $n + 2$ polynomial.
- Find a particular solution to $y'' + y = xe^{3x}$. In this case

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial. If both are zero, then a degree $n + 2$ polynomial.
- Find a particular solution to $y'' + y = xe^{3x}$. In this case $m = 3$ and hence

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial. If both are zero, then a degree $n + 2$ polynomial.
- Find a particular solution to $y'' + y = xe^{3x}$. In this case $m = 3$ and hence $2m + P = 6$ and $m^2 + Pm + Q = 10$.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial. If both are zero, then a degree $n + 2$ polynomial.
- Find a particular solution to $y'' + y = xe^{3x}$. In this case $m = 3$ and hence $2m + P = 6$ and $m^2 + Pm + Q = 10$. So let's try $u = Ax + B$.

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial. If both are zero, then a degree $n + 2$ polynomial.
- Find a particular solution to $y'' + y = xe^{3x}$. In this case $m = 3$ and hence $2m + P = 6$ and $m^2 + Pm + Q = 10$. So let's try $u = Ax + B$. It turns out that

Other methods in special cases: $R = p(x)e^{mx}$

- Suppose $R = p(x)e^{mx}$ where $m \in \mathbb{R}$ and $p(x)$ is a polynomial of degree n.
- We first try $y(x) = u(x)e^{mx}$. Then the ODE for u is $u'' + (2m + P)u' + (m^2 + Pm + Q)u = p(x)$. This equation is akin to the first case and hence we can try a polynomial for u. Clearly if $m^2 + Pm + Q \neq 0$ we ought to try a degree n polynomial. If it is 0 and $2m + P$ is not zero, then a degree $n + 1$ polynomial. If both are zero, then a degree $n + 2$ polynomial.
- Find a particular solution to $y'' + y = xe^{3x}$. In this case $m = 3$ and hence $2m + P = 6$ and $m^2 + Pm + Q = 10$. So let's try $u = Ax + B$. It turns out that $y_1 = \frac{e^{3x}(5x-3)}{50}$.

Examples from mechanics

Examples from mechanics

- Suppose a particle

Examples from mechanics

- Suppose a particle of mass 1 is under

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it?

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$,

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2x$.
- Thus $x'' = -k^2x$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2x$.
- Thus $x'' = -k^2x$. This equation can be solved to yield

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2x$.
- Thus $x'' = -k^2x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance?

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2x$.
- Thus $x'' = -k^2x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2 x = 0$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2 x = 0$. Further, what if

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2 x = 0$. Further, what if we apply another

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2 x = 0$. Further, what if we apply another external time-dependent force $F(t)$ on it?

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2 x = 0$. Further, what if we apply another external time-dependent force $F(t)$ on it? Then $x'' + 2cx' + k^2 x = F(t)$.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2x$.
- Thus $x'' = -k^2x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2x = 0$. Further, what if we apply another external time-dependent force $F(t)$ on it? Then $x'' + 2cx' + k^2x = F(t)$.
- This equation is precisely

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2 x = 0$. Further, what if we apply another external time-dependent force $F(t)$ on it? Then $x'' + 2cx' + k^2 x = F(t)$.
- This equation is precisely what we have been dealing with.

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2 x = 0$. Further, what if we apply another external time-dependent force $F(t)$ on it? Then $x'' + 2cx' + k^2 x = F(t)$.
- This equation is precisely what we have been dealing with. The key lies in finding

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2 x$.
- Thus $x'' = -k^2 x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2 x = 0$. Further, what if we apply another external time-dependent force $F(t)$ on it? Then $x'' + 2cx' + k^2 x = F(t)$.
- This equation is precisely what we have been dealing with. The key lies in finding solutions to

Examples from mechanics

- Suppose a particle of mass 1 is under the influence of a potential $V(x)$. Suppose it is at stable equilibrium at $x = 0$. What will happen if we perturb it? $V(x) \approx V(0) + \frac{k^2}{2}x^2$. Since $F = -V'$, $F = -k^2x$.
- Thus $x'' = -k^2x$. This equation can be solved to yield $x = A\cos(kt) + B\sin(kt)$. This is a Harmonic Oscillator.
- What if the particle is subject to air resistance? Stokes' law says that the viscous drag is $F = -2cx'$.
- Thus $x'' + 2cx' + k^2x = 0$. Further, what if we apply another external time-dependent force $F(t)$ on it? Then $x'' + 2cx' + k^2x = F(t)$.
- This equation is precisely what we have been dealing with. The key lies in finding solutions to the homogeneous problem.