Lecture 20 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

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Recap

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- The method of variation of parameters

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- Ended with the formulation of a damped forced oscillator:

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- The method of variation of parameters to solve second-order linear inhomogeneous ODE with constant coefficients. Discussed the Wronskian along the way.
- A simpler method when the RHS is $p(x)$ or $p(x)e^{mx}$.
- Ended with the formulation of a damped forced oscillator: $y'' + 2cy' + k^2y = F(t)$ where $c > 0, k \in \mathbb{R}$.

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