Lecture 21 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

IISc

Recap

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- Solved the damped oscillator

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- Solved the damped oscillator and RLC circuits.
- Defined open balls, interior points, and open sets. Gave examples and non-examples.

Exterior points, boundary points, and closed sets

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- One can prove (HW) that a set $S \subset \mathbb{R}^n$ is closed iff $S = int(S) \cup \partial S$.

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