Lecture 13 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

IISc

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- Similar matrices and determinants of linear maps $T : V \to V$.

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- Let $v_n = \begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix}$, and $M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Then $v_n = M v_{n-1}$. Thus $v_n = M^{n-2} v_2$. How does one write a formula for M^{n-2}?

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- So what is M^n? (This simple model is an example of a Markov chain.)

Similarity and powers

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- Conversely, if $\det(\lambda I - T) = 0$, then there exists a non-zero v such that $Tv = \lambda v$. Therefore, eigenvalues are precisely solutions to $p_T(\lambda) = \det(\lambda I - T) = 0$ lying in $\mathbb{F}$.
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