Lecture 14 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

IISc

Recap

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- We agreed that

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- We agreed that calculating M^n

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Simple criteria for checking similarity

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- So we find the eigenspaces by solving $Tv = \lambda v$.

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- For $\lambda = 1$

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- Likewise, the eigenspace of $\lambda = -1$ is spanned by $(0, 1, -1)$ and that of $\lambda = 3$ is spanned by $(2, 3, -1)$.

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Example-2

Example-2

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- Thus $P^{-1}TP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ where $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ -1 & -1 & 3 \end{bmatrix}$.

Example-3

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- Hence T is

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- Hence T is NOT diagonalisable.