HW 10 (to be tested on June 3)

1. Complete the proof in the class of the second derivative test for local minima and saddle points.

2. Let $f(x, y) = (3-x)(3-y)(x+y-3)$ on *all* of $\mathbb{R}^2$. Find its local extrema and classify them (if the second derivative test is applicable). Does f have global extrema?

3. Verify that $f(x, y, z) = x^4 + y^4 + z^4 - 4xyz$ has a critical point at $(1, 1, 1)$ and determine the nature of this critical point and determine its nature.

4. Parametrise the curve formed by the intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and the cylinder $x^2 + y^2 = ax$ where $z \geq 0$ and $a > 0$, i.e., give a piecewise C^1 path traversing the curve in a $1-1$ manner (except possibly at finitely many values of t) such that it appears clockwise when viewed from high above the xy-plane. Calculate the work done by (y^2, z^2, x^2) along this curve.