HW 9 (to be tested on May 27)

1. Prove that a vector field $\vec{F}$ is differentiable at a point $\vec{a}$ if and only if its component scalar fields are so. Moreover, prove that $\nabla_{\vec{v}}\vec{F} = (\nabla_{\vec{v}}F_1, \ldots)$.

2. Find a constant c so that at any point of intersection of the two spheres, $(x-c)^2 + y^2 + z^2 = 3$ and $x^2 + (y-1)^2 + z^2 = 1$, the corresponding tangent planes will be perpendicular to each other.

3. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is a C^2 function. Let $x = r\cos(\theta)$, $y = r\sin(\theta)$ where $0 < r < \infty, 0 < \theta < 2\pi$. Prove that $\vec{g}(r,\theta) = (x,y)$ is a C^2 function, that the composition $h(r,\theta) = f(x(r,\theta), y(r,\theta))$ is C^2, and that $f_{xx} + f_{yy} = f_{rr} + \frac{1}{r}f_r + \frac{1}{r^2}f_{\theta\theta}$.

4. Let $f(x,y) = \int_0^{\sqrt{xy}} e^{-t^2}dt$ for $x > 0, y > 0$. Prove that f is differentiable and compute f_x in terms of x, y.