Lecture 35 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

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Recap

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- Partitions of rectangles,

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- Partitions of rectangles, step functions,

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- Partitions of rectangles, step functions, their double integrals,

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- Proof: Recall that the boundary of a Type-I domain has zero area. Thus the extension $\tilde{f}$ to $[a,b] \times [c,d]$ is integrable.

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- Proof: Recall that the boundary of a Type-I domain has zero area. Thus the extension $\tilde{f}$ to $[a,b] \times [c,d]$ is integrable. Moreover, for each x, $\int_c^d \tilde{f}(x,y)dy$ is integrable because there are at most two discontinuities. Thus by Fubini, $\int\int_S f :=$ $\int\int_Q \tilde{f} = \int_a^b \int_c^d \tilde{f}(x,y)dydx = \int_a^b \int_{\phi_1(x)}^{\phi_2(x)} f(x,y)dydx$.

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- Example 1: Calculate the volume of an ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. An ellipsoid is a Type-I domain in 3-D:

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