Lecture 37 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

IISc

Recap

Recap

- Area enclosed by a Jordan curve

Recap

- Area enclosed by a Jordan curve using Green's theorem.

Recap

- Area enclosed by a Jordan curve using Green's theorem.
- Change of variables formula.

Recap

- Area enclosed by a Jordan curve using Green's theorem.
- Change of variables formula.
- Examples, including

Recap

- Area enclosed by a Jordan curve using Green's theorem.
- Change of variables formula.
- Examples, including the volume of a ball.

Parametrisation of surfaces

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def:

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u, v) : T \to \mathbb{R}^3$ that

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u, v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u, v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example:

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$.

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere.

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T.

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e.,

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary".

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand,

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u, v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u, v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u, v) \in T = [0, \pi] \times [0, 2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand, if $T = [0, \frac{\pi}{2}] \times [0, 2\pi]$ then it is a hemisphere whose

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u, v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u, v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u, v) \in T = [0, \pi] \times [0, 2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand, if $T = [0, \frac{\pi}{2}] \times [0, 2\pi]$ then it is a hemisphere whose boundary is a circle.

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand, if $T = [0,\frac{\pi}{2}] \times [0,2\pi]$ then it is a hemisphere whose boundary is a circle.
- Def:

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand, if $T = [0,\frac{\pi}{2}] \times [0,2\pi]$ then it is a hemisphere whose boundary is a circle.
- Def: A closed parametrised surface

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand, if $T = [0, \frac{\pi}{2}] \times [0, 2\pi]$ then it is a hemisphere whose boundary is a circle.
- Def: A closed parametrised surface is a set such that near

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand, if $T = [0,\frac{\pi}{2}] \times [0,2\pi]$ then it is a hemisphere whose boundary is a circle.
- Def: A closed parametrised surface is a set such that near every point it is

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand, if $T = [0,\frac{\pi}{2}] \times [0,2\pi]$ then it is a hemisphere whose boundary is a circle.
- Def: A closed parametrised surface is a set such that near every point it is a parametrised surface.

Parametrisation of surfaces

- Let $T \subset \mathbb{R}^2$ be a bounded region whose boundary is a piecewise C^1 Jordan curve (i.e., simple closed regular curve).
- Def: A parametrised surface is (the range of) a piecewise C^1 map $\vec{r}(u,v) : T \to \mathbb{R}^3$ that is $1-1$ on the interior.
- Example: $\vec{r}(u,v) = (\sin(u)\cos(v), \sin(u)\sin(v), \cos(u))$ where $(u,v) \in T = [0,\pi] \times [0,2\pi]$. The image is the unit sphere. The map is not $1-1$ on the boundary of T. This surface is a "closed" surface, i.e., it has no "boundary". On the other hand, if $T = [0, \frac{\pi}{2}] \times [0, 2\pi]$ then it is a hemisphere whose boundary is a circle.
- Def: A closed parametrised surface is a set such that near every point it is a parametrised surface. So the sphere is a closed parametrised surface.

Some more examples and infinitesimal area

Some more examples and infinitesimal area

- Example:

Some more examples and infinitesimal area

- Example: $\vec{r}(u,v) = (\cos(u), \sin(u), v)$ where $(u,v) \in [0,2\pi] \times [0,1]$.

Some more examples and infinitesimal area

- Example: $\vec{r}(u,v) = (\cos(u), \sin(u), v)$ where $(u,v) \in [0, 2\pi] \times [0,1]$. It is a right-circular cylinder

Some more examples and infinitesimal area

- Example: $\vec{r}(u, v) = (\cos(u), \sin(u), v)$ where $(u, v) \in [0, 2\pi] \times [0, 1]$. It is a right-circular cylinder with boundary being two circles. (

Some more examples and infinitesimal area

- Example: $\vec{r}(u, v) = (\cos(u), \sin(u), v)$ where $(u, v) \in [0, 2\pi] \times [0, 1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)

Some more examples and infinitesimal area

- Example: $\vec{r}(u, v) = (\cos(u), \sin(u), v)$ where $(u, v) \in [0, 2\pi] \times [0, 1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example:

Some more examples and infinitesimal area

- Example: $\vec{r}(u,v) = (\cos(u), \sin(u), v)$ where $(u,v) \in [0, 2\pi] \times [0,1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u,v) = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0, 2\pi] \times [0,1]$

Some more examples and infinitesimal area

- Example: $\vec{r}(u,v) = (\cos(u), \sin(u), v)$ where $(u,v) \in [0, 2\pi] \times [0,1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u,v) = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0, 2\pi] \times [0,1]$ is a right-circular cone with cone angle α.

Some more examples and infinitesimal area

- Example: $\vec{r}(u, v) = (\cos(u), \sin(u), v)$ where $(u, v) \in [0, 2\pi] \times [0, 1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u, v) = (v \sin(\alpha) \cos(u), v \sin(\alpha) \sin(u), v \cos(\alpha))$ where $(u, v) \in [0, 2\pi] \times [0, 1]$ is a right-circular cone with cone angle α. It is a (non-closed) parametrised surface

Some more examples and infinitesimal area

- Example: $\vec{r}(u,v) = (\cos(u), \sin(u), v)$ where $(u,v) \in [0,2\pi] \times [0,1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u,v) = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0,2\pi] \times [0,1]$ is a right-circular cone with cone angle α. It is a (non-closed) parametrised surface with boundary as a circle.

Some more examples and infinitesimal area

- Example: $\vec{r}(u, v) = (\cos(u), \sin(u), v)$ where $(u, v) \in [0, 2\pi] \times [0, 1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u, v) = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u, v) \in [0, 2\pi] \times [0, 1]$ is a right-circular cone with cone angle α. It is a (non-closed) parametrised surface with boundary as a circle.
- Suppose *dudv* is an infinitesimal area element

Some more examples and infinitesimal area

- Example: $\vec{r}(u,v) = (\cos(u), \sin(u), v)$ where $(u,v) \in [0,2\pi] \times [0,1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u,v) = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0,2\pi] \times [0,1]$ is a right-circular cone with cone angle α. It is a (non-closed) parametrised surface with boundary as a circle.
- Suppose $dudv$ is an infinitesimal area element in the $u - v$ plane.

Some more examples and infinitesimal area

- Example: $\vec{r}(u, v) = (\cos(u), \sin(u), v)$ where $(u, v) \in [0, 2\pi] \times [0, 1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u, v) = (v \sin(\alpha) \cos(u), v \sin(\alpha) \sin(u), v \cos(\alpha))$ where $(u, v) \in [0, 2\pi] \times [0, 1]$ is a right-circular cone with cone angle α. It is a (non-closed) parametrised surface with boundary as a circle.
- Suppose $dudv$ is an infinitesimal area element in the $u - v$ plane. Then the parallelogram formed

Some more examples and infinitesimal area

- Example: $\vec{r}(u, v) = (\cos(u), \sin(u), v)$ where $(u, v) \in [0, 2\pi] \times [0, 1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u, v) = (v \sin(\alpha) \cos(u), v \sin(\alpha) \sin(u), v \cos(\alpha))$ where $(u, v) \in [0, 2\pi] \times [0, 1]$ is a right-circular cone with cone angle α. It is a (non-closed) parametrised surface with boundary as a circle.
- Suppose $dudv$ is an infinitesimal area element in the $u - v$ plane. Then the parallelogram formed in $\mathbb{R}^3$ has sides

Some more examples and infinitesimal area

- Example: $\vec{r}(u,v) = (\cos(u), \sin(u), v)$ where $(u,v) \in [0,2\pi] \times [0,1]$. It is a right-circular cylinder with boundary being two circles. (It is not a closed parametrised surface.)
- Example: $\vec{r}(u,v) = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0,2\pi] \times [0,1]$ is a right-circular cone with cone angle α. It is a (non-closed) parametrised surface with boundary as a circle.
- Suppose $dudv$ is an infinitesimal area element in the $u-v$ plane. Then the parallelogram formed in $\mathbb{R}^3$ has sides $\vec{r}_u du = du(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u})$ and $\vec{r}_v dv = dv(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v})$.

Regular/Smooth surfaces

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves,

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone,

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example:

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function,

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance,

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| du dv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is not differentiable at $x = y = 0$.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is not differentiable at $x = y = 0$. The problem here lies

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is not differentiable at $x = y = 0$. The problem here lies with this *particular* parametrisation because

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is not differentiable at $x = y = 0$. The problem here lies with this *particular* parametrisation because we already saw the the sphere

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is not differentiable at $x = y = 0$. The problem here lies with this *particular* parametrisation because we already saw the the sphere can be parametrised even near

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is not differentiable at $x = y = 0$. The problem here lies with this *particular* parametrisation because we already saw the the sphere can be parametrised even near the equator as a regular surface.

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is not differentiable at $x = y = 0$. The problem here lies with this *particular* parametrisation because we already saw the the sphere can be parametrised even near the equator as a regular surface. So the choice of

Regular/Smooth surfaces

- The infinitesimal area is $\|\vec{r}_u \times \vec{r}_v\| dudv$. Just as for regular curves, we define regular surfaces as those with $\vec{r}_u \times \vec{r}_v \neq 0$ everywhere. Such surfaces are also called "smooth".
- For a cone, $\vec{r}_u = v(-\sin(\alpha)\sin(u), \sin(\alpha)\cos(u), 0)$ and $\vec{r}_v = (\sin(\alpha)\cos(u), \sin(\alpha)\sin(u), \cos(\alpha))$. Thus $\vec{r}_u \times \vec{r}_v = \vec{0}$ when $v = 0$. The vertex of the cone is not a smooth point.
- Example: Suppose $z = f(x, y)$ where f is a C^1 function, $\vec{r}(u, v) = (u, v, f(u, v))$. Then $\vec{r}_u = (1, 0, f_u)$ and $\vec{r}_v = (0, 1, f_v)$. Thus $\vec{r}_u \times \vec{r}_v = (-f_u, -f_v, 1)$ and is hence never zero. So the graph of a C^1 function is a regular parameterised surface.
- For instance, if we take $z = \sqrt{1 - x^2 - y^2}$, the function is not differentiable at $x = y = 0$. The problem here lies with this *particular* parametrisation because we already saw the the sphere can be parametrised even near the equator as a regular surface. So the choice of a parametrisation is important.

Normals and areas

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface.

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p.

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces.

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways:

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$.

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph,

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph, $F(x, y, z) = z - f(x, y)$ gives the same

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x,y,z) = 0$ and as $(x(u,v), y(u,v), z(u,v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph, $F(x,y,z) = z - f(x,y)$ gives the same unit normal as the previous one.

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x,y,z) = 0$ and as $(x(u,v), y(u,v), z(u,v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph, $F(x,y,z) = z - f(x,y)$ gives the same unit normal as the previous one.
- The infinitesimal area vector

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph, $F(x, y, z) = z - f(x, y)$ gives the same unit normal as the previous one.
- The infinitesimal area vector is $d\vec{A} = \vec{r}_u \times \vec{r}_v dudv$.

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph, $F(x, y, z) = z - f(x, y)$ gives the same unit normal as the previous one.
- The infinitesimal area vector is $d\vec{A} = \vec{r}_u \times \vec{r}_v dudv$.
- The area of a parametric surface is

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph, $F(x, y, z) = z - f(x, y)$ gives the same unit normal as the previous one.
- The infinitesimal area vector is $d\vec{A} = \vec{r}_u \times \vec{r}_v dudv$.
- The area of a parametric surface is $\int\int_T \|\vec{r}_u \times \vec{r}_v\| dudv$. For instance,

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph, $F(x, y, z) = z - f(x, y)$ gives the same unit normal as the previous one.
- The infinitesimal area vector is $d\vec{A} = \vec{r}_u \times \vec{r}_v dudv$.
- The area of a parametric surface is $\int\int_T \|\vec{r}_u \times \vec{r}_v\| dudv$. For instance, for a graph $z = f(x, y)$, the area is

Normals and areas

- $\vec{r}_u \times \vec{r}_v$ is normal to the surface. Indeed, let $(u(t), v(t))$ be a C^1 path on the surface passing through p. Then $\frac{d\vec{r}}{dt} = \vec{r}_u u' + \vec{r}_v v'$. Thus $\vec{r}'$ is perpendicular to $\vec{r}_u \times \vec{r}_v$.
- This is another way to study normals to surfaces. The same surface can be given in two ways: $F(x, y, z) = 0$ and as $(x(u, v), y(u, v), z(u, v))$. The normals are ∇F and $\vec{r}_u \times \vec{r}_v$. For instance, even for a graph, $F(x, y, z) = z - f(x, y)$ gives the same unit normal as the previous one.
- The infinitesimal area vector is $d\vec{A} = \vec{r}_u \times \vec{r}_v dudv$.
- The area of a parametric surface is $\int\int_T \|\vec{r}_u \times \vec{r}_v\| dudv$. For instance, for a graph $z = f(x, y)$, the area is $\int\int \sqrt{1 + f_x^2 + f_y^2} dxdy$.

An example and scalar line integrals

An example and scalar line integrals

- Hemisphere:

An example and scalar line integrals

- Hemisphere:$\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$.

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore,

An example and scalar line integrals

- Hemisphere:$\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression:

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path,

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x, y, z)$ is a bounded function along the path,

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x,y,z)$ is a bounded function along the path, then $\int fds = \int_a^b f(\gamma(t))\|\gamma'(t)\|dt$ is called the scalar line integral of f along γ. (

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x,y,z)$ is a bounded function along the path, then $\int f ds = \int_a^b f(\gamma(t))\|\gamma'(t)\|dt$ is called the scalar line integral of f along γ. (For instance, the charge on a wire.)

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x,y,z)$ is a bounded function along the path, then $\int fds = \int_a^b f(\gamma(t))\|\gamma'(t)\|dt$ is called the scalar line integral of f along γ. (For instance, the charge on a wire.)
- A crucial point:

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x, y, z)$ is a bounded function along the path, then $\int f ds = \int_a^b f(\gamma(t))\|\gamma'(t)\| dt$ is called the scalar line integral of f along γ. (For instance, the charge on a wire.)
- A crucial point: The scalar line integral is

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x,y,z)$ is a bounded function along the path, then $\int f ds = \int_a^b f(\gamma(t))\|\gamma'(t)\|dt$ is called the scalar line integral of f along γ. (For instance, the charge on a wire.)
- A crucial point: The scalar line integral is reparametrisation invariant:

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x, y, z)$ is a bounded function along the path, then $\int f ds = \int_a^b f(\gamma(t))\|\gamma'(t)\|dt$ is called the scalar line integral of f along γ. (For instance, the charge on a wire.)
- A crucial point: The scalar line integral is reparametrisation invariant: If $t(\tau)$ is a reparametrisation,

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x,y,z)$ is a bounded function along the path, then $\int f ds = \int_a^b f(\gamma(t))\|\gamma'(t)\| dt$ is called the scalar line integral of f along γ. (For instance, the charge on a wire.)
- A crucial point: The scalar line integral is reparametrisation invariant: If $t(\tau)$ is a reparametrisation, then $\|\frac{d\gamma}{d\tau}\| = \|\gamma'(t)\||\frac{dt}{d\tau}|$. Thus

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r_u} = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r_v} = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r_u} \times \vec{r_v}\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x,y,z)$ is a bounded function along the path, then $\int fds = \int_a^b f(\gamma(t))\|\gamma'(t)\|dt$ is called the scalar line integral of f along γ. (For instance, the charge on a wire.)
- A crucial point: The scalar line integral is reparametrisation invariant: If $t(\tau)$ is a reparametrisation, then $\|\frac{d\gamma}{d\tau}\| = \|\gamma'(t)\||\frac{dt}{d\tau}|$. Thus the "new" integral is $\int_{[c,d]} f(\gamma(t(\tau)))\|\gamma'(t)\||\frac{dt}{d\tau}|d\tau$ which by

An example and scalar line integrals

- Hemisphere: $\vec{r} = (r\sin(u)\cos(v), r\sin(u)\sin(v), r\cos(u))$ and hence $\vec{r}_u = (r\cos(u)\cos(v), r\cos(u)\sin(v), -r\sin(u))$, $\vec{r}_v = (-r\sin(u)\sin(v), r\sin(u)\cos(v), 0)$. So $\|\vec{r}_u \times \vec{r}_v\| = r^2\sin(v)$. Therefore, the area is $2\pi r^2$.
- A digression: If $\vec{\gamma}(t)$ is a piecewise C^1 regular path, and $f(x,y,z)$ is a bounded function along the path, then $\int f ds = \int_a^b f(\gamma(t))\|\gamma'(t)\|dt$ is called the scalar line integral of f along γ. (For instance, the charge on a wire.)
- A crucial point: The scalar line integral is reparametrisation invariant: If $t(\tau)$ is a reparametrisation, then $\|\frac{d\gamma}{d\tau}\| = \|\gamma'(t)\||\frac{dt}{d\tau}|$. Thus the "new" integral is $\int_{[c,d]} f(\gamma(t(\tau)))\|\gamma'(t)\||\frac{dt}{d\tau}|d\tau$ which by the change of variables formula equals the "old" integral.

Scalar surface integral and examples

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S.

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v))\|\vec{r}_u \times \vec{r}_v\| dudv$.
- When $f = 1$,

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S fdA := \int\int_T f(\vec{r}(u,v))\|\vec{r}_u \times \vec{r}_v\|dudv$.
- When $f = 1$, we get the area. (Akin to the length

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S fdA := \int\int_T f(\vec{r}(u,v))\|\vec{r}_u \times \vec{r}_v\| dudv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S fdA := \int\int_T f(\vec{r}(u,v))\|\vec{r}_u \times \vec{r}_v\|dudv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass:

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v))\|\vec{r}_u \times \vec{r}_v\| dudv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density,

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density, then $x_{CM} = \int\int x f dA$ and likewise for other coordinates.

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S fdA := \int\int_T f(\vec{r}(u,v))\|\vec{r}_u \times \vec{r}_v\| dudv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density, then $x_{CM} = \int\int xfdA$ and likewise for other coordinates. For instance, for a cone $\vec{r} = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0,2\pi] \times [0,l]$, we see that $dA = v\sin(\alpha)dudv$.

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v))\|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density, then $x_{CM} = \int\int x f dA$ and likewise for other coordinates. For instance, for a cone $\vec{r} = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0,2\pi] \times [0,l]$, we see that $dA = v\sin(\alpha) du dv$. Thus if $f = 1$ (uniform density),

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density, then $x_{CM} = \int\int x f dA$ and likewise for other coordinates. For instance, for a cone $\vec{r} = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0,2\pi] \times [0,l]$, we see that $dA = v\sin(\alpha) du dv$. Thus if $f = 1$ (uniform density), then $\int\int z dA = \int_0^l \int_0^{2\pi} v^2 \cos(\alpha)\sin(\alpha) du dv = \frac{\pi}{3}\sin(2\alpha)$.

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int \int_S f dA := \int \int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density, then $x_{CM} = \int \int x f dA$ and likewise for other coordinates. For instance, for a cone $\vec{r} = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0, 2\pi] \times [0, l]$, we see that $dA = v\sin(\alpha) du dv$. Thus if $f = 1$ (uniform density), then $\int \int z dA = \int_0^l \int_0^{2\pi} v^2 \cos(\alpha)\sin(\alpha) du dv = \frac{\pi}{3}\sin(2\alpha)$. It is easy to see that

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density, then $x_{CM} = \int\int x f dA$ and likewise for other coordinates. For instance, for a cone $\vec{r} = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0, 2\pi] \times [0, l]$, we see that $dA = v\sin(\alpha) du dv$. Thus if $f = 1$ (uniform density), then $\int\int z dA = \int_0^l \int_0^{2\pi} v^2 \cos(\alpha)\sin(\alpha) du dv = \frac{\pi}{3}\sin(2\alpha)$. It is easy to see that $x_{CM} = y_{CM} = 0$.

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density, then $x_{CM} = \int\int x f dA$ and likewise for other coordinates. For instance, for a cone $\vec{r} = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0,2\pi] \times [0,l]$, we see that $dA = v\sin(\alpha) du dv$. Thus if $f = 1$ (uniform density), then $\int\int z dA = \int_0^l \int_0^{2\pi} v^2 \cos(\alpha)\sin(\alpha) du dv = \frac{\pi}{3}\sin(2\alpha)$. It is easy to see that $x_{CM} = y_{CM} = 0$. Thus the centre of mass

Scalar surface integral and examples

- Let $S = \vec{r}(T)$ be a parametrised surface and let f be a bounded scalar field on S. Then $\int\int_S f dA := \int\int_T f(\vec{r}(u,v)) \|\vec{r}_u \times \vec{r}_v\| du dv$.
- When $f = 1$, we get the area. (Akin to the length of a regular curve in the case of line integrals.)
- Centre of mass: If f is the density, then $x_{CM} = \int\int x f dA$ and likewise for other coordinates. For instance, for a cone $\vec{r} = (v\sin(\alpha)\cos(u), v\sin(\alpha)\sin(u), v\cos(\alpha))$ where $(u,v) \in [0, 2\pi] \times [0, l]$, we see that $dA = v\sin(\alpha) du dv$. Thus if $f = 1$ (uniform density), then $\int\int z dA = \int_0^l \int_0^{2\pi} v^2 \cos(\alpha)\sin(\alpha) du dv = \frac{\pi}{3}\sin(2\alpha)$. It is easy to see that $x_{CM} = y_{CM} = 0$. Thus the centre of mass can lie outside the surface.