Lecture 6 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

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Recap

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Lecture 6

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- Moreover, if a left inverse exists, the right inverse is THE left inverse, i.e., the right inverse is unique. (Indeed, $TR_1 = TR_2 = I$ and hence $LTR_1 = LTR_2 \Rightarrow R_1 = R_2$.

Lecture 6

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Lecture 6

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Solving linear equations

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Row-echelon form

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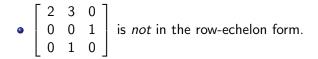
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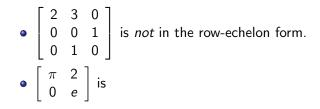
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- The point is to solve the *last* non-trivial equation and back-substitute to solve the rest.







•
$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$
 is *not* in the row-echelon form.
• $\begin{bmatrix} \pi & 2 \\ 0 & e \end{bmatrix}$ is in the row-echelon form.

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