Lecture 7 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

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The row-reduction algorithm

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- Identify the left-most pivot among all rows. Suppose it occurs in the i^{th} row.
- Interchanging rows, make sure that R_i is the first row.
- Divide out the first-row pivot to make it 1.
- "Clear" everything below the first-row pivot

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- By induction/recursion/"Rinse and repeat"

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- By induction/recursion/"Rinse and repeat" the $(m-1) \times n$ matrix of the next $m-1$ rows can be assumed to be in the required form.
- Clear the elements in the first row using the pivots in the other rows. (On a computer, you can implement it iteratively or recursively.)

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- To solve $Ax = b$, consider the augmented matrix $[A|b]$, and row-reduce it to its RREF $[\tilde{A}|\tilde{b}]$.
- If any row of $\tilde{A}$ is 0, but the corresponding entry of b is not, then the system is inconsistent.

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- If any row of $\tilde{A}$ is 0, but the corresponding entry of b is not, then the system is inconsistent.
- If it is consistent, starting from the bottom of $\tilde{A}$ solve for the first non-zero pivoted variable.
- Inductively/recursively, solve for the other pivoted variables.

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- Using the nullity-rank theorem

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- Using the nullity-rank theorem one can prove that the row rank of C equals its *column rank* (the dimension of the column space/the number of non-zero columns in the Column reduced Column-echelon form/the number of non-zero rows in the RREF of A^T).

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Example-1

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Example-2

Example-2

- Solve:

Example-2

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Example-2

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Example-2

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Example-2

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Example-2

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