Lecture 8 - UM 102 (Spring 2021)

Vamsi Pritham Pingali

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Recap

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- If A is (left-)invertible: There is a matrix B such that $BA = I$. Hence, the corresponding linear map $\tilde{T}$ satisfies $\tilde{T}T = I$. Since a left inverse is *the* inverse, $T\tilde{T} = I$. (As a consequence, the left inverse of the *matrix* A is its right inverse.) □

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