Lecture 11 - UM 102 (Spring 2021)

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IISc

Recap

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Recap

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- Proved the expansion-along-the-first-column property

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- Proved the expansion-along-the-first-column property assuming existence. Hence expansion-along-any-column.
- Proved existence and by construction, the expansion-along-any-row-property.

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- Each time we scale a row by a constant c_i the determinant scales and each row-exchange leads to a -1.
- So $\det(A) = \frac{(-1)^p \det(U)}{c_1 c_2 \ldots}$.

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- If A, B are two $n \times n$ matrices, then $\det(AB) = \det(A)\det(B)$. This formula is *extremely* important.
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- One naive thing to do is to consider an ordered basis $e_1, \ldots, e_n$ for both, the domain and the target. Then T is represented by a matrix $[T]$. We can attempt to define $\det(T)$ as $\det([T])$. However, what happens when we change the ordered basis?
- Let $e'_1, e'_2, \ldots$ be a new ordered basis. Recall that $e'_i = \sum_j P_{ji} e_j$ for some $P_{ji} \in \mathbb{F}$. Also recall that in an ordered basis the first column of the matrix associated to T is simply the component vector of $T(e'_1)$ and likewise

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