UM 102 - Lecture 1

Vamsi Pritham Pingali

IISc

Logistics

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- Office : N23 (Office hours during the period of online classes : Tue : 10:30-11:30 on MS Teams), Email : vamsipingali@iisc.ac.in

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- Text book : Apostol, Calculus (Vol 2).

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- $m \times n$ matrices with complex/real entries.
- The set of all differentiable functions $x, y : \mathbb{R} \to \mathbb{R}$ satisfying $x' = 2x + 3y, y' = 4x + 5y$ form a vector space over $\mathbb{R}$.

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- Often, one considers an *ordered* basis, i.e., a basis written in a specified order. In that case, every vector $v = \sum_k c_k e_k$. The (uniquely determined) numbers c_k are called *components* of v relative to the ordered basis $\{e_1, \ldots, e_n\}$.