Lecture 23 - UM 102 (Spring 2021)

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IISc

Recap

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- Limit and continuity laws (

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- Motivated the consideration of interior points for talking of derivatives.

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- If $f(\vec{x}) = \|\vec{x}\|^2$, compute $\nabla_{\vec{v}} f(\vec{a})$:

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- If $f(\vec{x}) = \|\vec{x}\|^2$, compute $\nabla_{\vec{v}} f(\vec{a})$: $g(t) = \|\vec{a}\|^2 + t^2\|\vec{v}\|^2 + 2t\langle \vec{a}, \vec{v} \rangle$.

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