HW 2

1. Prove that an onto function $T : V \to W$ has a left inverse if and only if it is $1-1$.

2. Let W be a vector space over a field $\mathbb{F}$. Let V be a finite-dimensional vector space over $\mathbb{F}$ with $\dim(V) = n$, and $T : V \to W$ be an onto linear map. Then prove that the following statements are equivalent to one another.

   (a) T is $1-1$.

   (b) If $e_1, \ldots, e_p$ are linearly independent in V, then $T(e_1), \ldots, T(e_p)$ are so in W.

   (c) $\dim(W) = n$.

   (d) If $e_1, \ldots, e_n$ is a basis for V, then $T(e_1), \ldots, T(e_n)$ is so for W.

3. Let V be a vector space and $S \subseteq V$ be a *subset*. Prove that $S^{\perp}$ is a subspace.

4. Consider $\mathbb{R}^3$ with the usual inner product. Find an orthonormal basis for the subspace spanned by $u = (1, 1, 1), v = (1, 0, -1), w = (3, 2, 1)$.

5. Consider the real vector space of real-valued continuous functions on $[-1, 1]$. Define the inner product $\langle f, g \rangle = \int_{-1}^{1} x^2 f(x) g(x) dx$.

   (a) Consider the subspace S spanned by the set $\{1, x, x^2, x^2 + x\}$. Find an orthonormal basis for S.

   (b) Find the best approximation of e^x from S.