HW 3

1. Prove that row-reduction does not change the row space. (A part of this problem is to make this statement rigorous.)

2. Prove that if F is a multilinear function, then $F(\ldots, v_k + c_1 w_1 + \ldots + c_m w_m, \ldots) = F(\ldots, v_k, \ldots) + c_1 F(\ldots, w_1, \ldots) + c_2 F(\ldots, w_2, \ldots) + \ldots$.

3. Assuming the expansion-along-the-first-row-or-first-column-property, state and prove the expansion-along-any-row-or-column-property for determinants.

4. Prove the following formula for the Vandermonde determinant:

$$\det \begin{pmatrix} 1 & x_1 & x_1^2 & \ldots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \ldots & x_2^{n-1} \\ \vdots & \vdots & \ddots & \ldots & \vdots \\ 1 & x_m & x_m^2 & \ldots & x_m^{n-1} \end{pmatrix} = \Pi_{1 \leq i \leq j \leq n} (x_j - x_i) \tag{1}$$