HW 4

1. Assuming the expansion-along-the-first-row-or-first-column-property, state and prove the expansion-along-any-row-or-column-property for determinants.

2. Consider the vector space V of real polynomials of degree ≤ 2 and the linear map $D : V \to V$ given by $Dp = p'$. Write the matrices D_1, D_2 of D in the ordered bases $1, x, x^2$ and $1, x+1, x^2+x$ respectively. Find an invertible matrix P such that $D_2 = P^{-1} D_1 P$.

3. Let $f_1, \ldots, f_n : \mathbb{R} \to \mathbb{R}$ be differentiable functions. Consider a real $n \times n$ matrix $A(x)$ whose first row is $[f_1(x)\ f_2(x)\ \ldots\ f_n(x)]$ and whose other rows are simply constant real numbers. Prove that the function $\det(A(x)) : \mathbb{R} \to \mathbb{R}$ is differentiable and that its derivative is $f_1'(x)M_{11} - f_2'(x)M_{12} + \ldots + (-1)^{n+1} f_n'(x)M_{1n}$ where M_{ij} is the ij^{th} minor.

4. If A is a square matrix, then prove that $p_A(\lambda) = \det(\lambda I - A)$ is a polynomial of degree n such that the coefficient of λ^{n-1} is $-\sum_i A_{ii}$. Prove that $\sum_i \lambda_i = \sum_i A_{ii}$ and that $\det(A) = \lambda_1 \lambda_2 \ldots$.