HW 6

1. (a) Prove that there is no scalar field f such that $\nabla_{\vec{v}} f(\vec{a}) > 0$ for a fixed vector $\vec{a}$ and every non-zero vector $\vec{v}$.

   (b) Give an example of a scalar field f such that $\nabla_{\vec{v}} f(\vec{a}) > 0$ for a fixed vector $\vec{v}$ and every vector $\vec{a}$.

2. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function. Prove that $\lim_{\vec{r}\to\vec{a}} f(\vec{r})$ exists and equals L if and only if for any sequence $\vec{r}_n \to \vec{a}$ with $\vec{r}_n \neq \vec{a} \; \forall \; n$ (meaning that the components converge individually to the respective components of $\vec{a}$), the sequence $f(\vec{r}_n)$ converges to L.

3. Prove that $\nabla_{\vec{v}} f(a, b)$ exists for all $\vec{v}, a, b$ and is linear in $\vec{v}$ for $f(x, y) = \frac{x^3 y}{x^4 + y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Also prove that f is continuous everywhere.