# HW 7

1. Let $f(x, y) = 3x^2 + y^2$. Why is $f$ differentiable on $\mathbb{R}^2$? Find the points $(x, y)$ and the directions for which the directional derivative of $f(x, y)$ has the largest value if the points $(x, y)$ are restricted to lie on the circle $x^2 + y^2 = 1$.

2. Prove that $f(x, y) = \tan\left(x^2 + y^2 + \frac{\pi}{2}\right)$ when $0 < x^2 + y^2 < 1$ and $f(0, 0) = 0$ is differentiable everywhere on $x^2 + y^2 < 1$ except at $(0, 0)$ (where you need to prove that it is *not* differentiable). Calculate its total derivative linear map at the point $(\frac{1}{2}, \frac{1}{2})$.

3. A differentiable *path* is a vector-valued function $\vec{r}(t) : (a, b) \subset \mathbb{R} \to \mathbb{R}^n$. A regular path is one whose velocity $\vec{r}'(t) \neq 0$ for all $t \in (a, b)$. A curve is the range of a path, i.e., a curve is a subset of $\mathbb{R}^n$. Suppose $\vec{u}(\tau) : (c, d) \to \mathbb{R}^n$ is another path such that it is a *reparametrisation* of $\vec{r}(t)$, i.e., $\tau(t) : (a, b) \to (c, d)$ is 1-1 onto continuously differentiable function such that its inverse is also continuously differentiable, $\tau'(t) > 0$ everywhere, and $\vec{u}(\tau(t)) = \vec{r}(t)$. Firstly, prove that the ranges of the paths, i.e., the curves are the same curve $C$. Prove that the directional derivative along $C$, i.e., $\frac{df}{ds}$ is the same at any point $(x_0, y_0)$ on $C$. In other words, $\frac{df}{ds}$ is a property of the *curve* (the geometric object) and not of any particle moving on it.

4. Find a constant $c$ so that at any point of intersection of the two spheres, $(x - c)^2 + y^2 + z^2 = 3$ and $x^2 + (y - 1)^2 + z^2 = 1$, the corresponding tangent planes will be perpendicular to each other.

5. Prove that if $\vec{a}$ is a point on a non-empty regular level set $f = c$ (where $f : \mathbb{R}^n \to \mathbb{R}$ is a $C^1$ function), the following hold.

   (a) The subspace of all $\vec{v}$ satisfying $\langle \nabla f(\vec{a}), \vec{v} \rangle = 0$ is $n - 1$-dimensional.

   (b) For every vector $\vec{v}$ such that $\langle \nabla f(\vec{a}), \vec{v} \rangle = 0$, there is a $C^1$ path $\vec{r} : (-1, 1) \to \mathbb{R}^n$ such that $f(\vec{r}(t)) = c \;\forall\; t$, $\vec{r}(0) = \vec{a}$ and $\vec{r}'(0) = \vec{v}$.