

HW 1(to be tested on Jan 12)

1. Given two non-empty sets A and B , construct the cartesian product $A \times B$ using ZF and define relations on $A \times B$ as well as the concept of a function from A to B . Give examples and non-examples.
2. Prove that the relation defined in the class to show that partitions come from equivalence relations, is an equivalence relation.
3. Complete Zagier's proof done in the class and show that every prime that is a sum of two squares is of the form $4k + 1$.
4. Prove that if $a > b$ then $-a < -b$ where $a, b \in \mathbb{Z}$.
5. Assuming that an infinite subset of a countable set is countable, prove that $\mathbb{N} \times \mathbb{N}$ is countable.