HW 3(to be tested on Feb 8)

1. Complete the proofs of the following.
   (a) "A connected graph G has a closed Eulerian trail iff all its vertices have even degree." (In the class we deleted two edges and still assumed the resulting graph was connected. Remove that assumption.)
   (b) "A connected simple graph is a tree iff for any pair of vertices there is exactly one path between them." (In the class we did not prove why the symmetric difference of two paths joining the same vertices is a disjoint union of cycles.)
   (c) The handshaking lemma for multigraphs.
   (d) Prove that $i \sim j$ if i and j are connected by a path is transitive.
   (e) Prove that path connected is equivalent to trail connected and to being walk connected.
   (f) Prove that the symmetric difference of two paths having the same endpoints is a union of disjoint cycles.
2. Let $z(n)$ be the number of simple graphs G on a vertex set $[n]$ in which no connected component has more than three vertices. Find the exponential generating function $\sum_{n\geq 0} z(n)\frac{x^n}{n!}$.