

## HW 4 (to be tested on Feb 15)

1. Solve Diophantine equations when  $a, b, e$  are not necessarily non-negative integers.
2. Show that  $\gcd(a, m) \leq \gcd(a, mn)$  for any non-negative integers  $a, m, n$ .
3. Prove that for natural numbers  $m, a, b > 0$ , the identity  $m\gcd(a, b) = \gcd(ma, mb)$  is satisfied.
4. You are given two hour glasses : a 6-minute hourglass and an 11-minute hourglass. How can you measure 13 minutes using them ?
5. Define the gcd of three natural numbers  $a, b, c$  and show that it is equal to  $\gcd(a, \gcd(b, c))$ . Also show that it is equal to  $ax + by + cz$  for three integers  $x, y, z$ .