HW 7 (to be tested on Mar 29)

1. Prove that every polynomial $p(x_1, \ldots, x_n) \in R[x_1, x_2, \ldots, x_n]$ where R is a commutative ring, can be uniquely written as a sum of homogeneous polynomials. Every symmetric polynomial can be be uniquely written as a sum of homogeneous symmetric polynomials.

2. Prove that the decomposition of a symmetric polynomial in terms of elementary symmetric ones is unique.

3. Assuming that rigid motions of $\mathbb{R}^2$ preserving the origin are of the form $\vec{r} \to [A]\vec{r}$ where $[A]$ is a matrix satisfying $[A]^T[A] = [A][A]^T = I$, prove that the group of rigid motions of a regular n-gon (with centre as the origin) is $D_n = \{1, r, r^2, \ldots, r^{n-1}, s, rs, r^2s, \ldots, r^{n-1}s | r^n = s^2 = 1, rs = sr^{-1}\}$.

4. Let G be an Abelian group and let H consist of those elements of G having finite order. Prove that H is a subgroup.

5. Prove that $\mathbb{Q}$ is not generated by a finite number of elements.