HW 2 (due on Feb 3, Friday, in the class)

1. (Rudin chapter 3 Problem 7) Prove that the convergence of $\sum a_n$ implies the convergence of $\sum \frac{\sqrt{a_n}}{n}$ if $a_n \geq 0$.

2. (Rudin chapter 3 problem 9)Find the radius of convergence of each of the following power series :

    (a) $\sum n^3 z^n$

    (b) $\sum \frac{2^n}{n!} z^n$

    (c) $\sum \frac{2^n}{n^2} z^n$

    (d) $\sum \frac{n^3}{3^n} z^n$.

3. (Rudin chapter 3 problem 12 part a)) Suppose $a_n > 0$ and $\sum a_n$ converges. Put $r_n = \sum_{m=n}^{\infty} a_m$. Prove that

$$\frac{a_m}{r_m} + \ldots + \frac{a_n}{r_n} > 1 - \frac{r_n}{r_m}$$

if $m < n$, and deduce that $\sum \frac{a_n}{r_n}$ diverges.