# HW 4 (due on 10th March in the class)

1. (Rudin chapter 4 problem 6) If $f : E \subset (X, d_X) \to (Y, d_Y)$ is a function, then the *graph* of $f$ is the set of points $(x, f(x))$ in the metric space $(X \times Y, d_{X\times Y}((x, y), (a, b)) = \sqrt{d_X(x, a)^2 + d_Y(a, b)^2})$. Suppose $E$ is a compact subset of $X$. Prove that $f$ is continuous on $E$ if and only if its graph is a compact subset of $X \times Y$.

2. (Rudin chapter 4 problem 12) A uniformly continuous function of a uniformly continuous function is uniformly continuous. State this more precisely and prove it.

3. (Rudin chapter 4 problem 15) Call a mapping of $X$ to $Y$ open if $f(V)$ is an open subset of $Y$ whenever $V$ is an open subset of $X$.
   Prove that every open continuous map from $\mathbb{R}$ to itself is monotonic.