HW 5 (due on 24th March in the class)

1. (Rudin chapter 5 problem 11, See the hint in Rudin) Suppose f is defined in a neighbourhood of x and suppose $f''(x)$ exists. Show that $\lim_{h\to 0} \frac{f(x+h)+f(x-h)-2f(x)}{h^2} = f''(x)$ and show by a counterexample that the limit may exist even if $f''(x)$ does not.

2. (Rudin chapter 5 problem 15, See the hint in Rudin) Suppose $a \in \mathbb{R}$, f is a twice-differentiable function on (a, ∞), and M_0, M_1, M_2 are the suprema of $|f|, |f'|, |f''|$ respectively on (a, ∞). Prove that $M_1^2 \leq 4M_0M_2$.

3. (Rudin chapter 6 problem 4) If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rational x, prove that f is not Riemann integrable on $[a, b]$ for any $a < b$.