HW 6 (due on 7th April in the class)

1. (Rudin chapter 6 problem 7) Suppose $f : [0,1] \to \mathbb{R}$ is Riemann integrable on $[c,1]$ for every $c > 0$. Define $\displaystyle\int_0^1 f dx = \lim_{c \to 0^+} \int_c^1 f dx$ if the limit exists and is finite.
   (a) If f is Riemann integrable on $[0,1]$ show that this definition of the integral agrees with the old one.
   (b) Construct a function such that the above limit exists, although it fails to exist with $|f|$ in the place of f.
2. (Rudin chapter 7 problem 6) Prove that $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval but does not converge absolutely for any value of x.
3. (Rudin chapter 7 problem 13 part a)) Assume that f_n is a sequence of monotonically increasing functions from $\mathbb{R} \to [0,1]$. Prove that there is a function f and a sequence n_k such that $f(x) = \lim_{k\to\infty} f_{n_k}(x)$ for every $x \in \mathbb{R}$.